

## 82 DERIVADAS (con SOLUCIONES)

■ Hallar las derivadas simplificadas de las siguientes funciones:

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|---|--|
| <p>1. <math>y = 5</math> <span style="float: right;"><math>(y'=0)</math></span></p>   | <p>20. <math>y = \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + 1</math> <span style="float: right;"><math>\left(y' = -\frac{3}{x^4} - \frac{2}{x^3} - \frac{1}{x^2}\right)</math></span></p>     |
| <p>2. <math>y = 3/2</math> <span style="float: right;"><math>(y'=0)</math></span></p>   | <p>21. <math>y = \frac{1}{x^2 + 2x - 3}</math> <span style="float: right;"><math>\left(y' = -\frac{2x + 2}{(x^2 + 2x - 3)^2}\right)</math></span></p>  |
| <p>3. <math>y = 3x</math> <span style="float: right;"><math>(y'=3)</math></span></p>  | <p>22. <math>y = \frac{3}{x^3 - 2x^2 + 5}</math> <span style="float: right;"><math>\left(y' = -3 \frac{3x^2 - 4x}{(x^3 - 2x^2 + 5)^2}\right)</math></span></p>                                   |
| <p>4. <math>y = 2x - 3</math> <span style="float: right;"><math>(y'=2)</math></span></p>  | <p>23. <math>y = \frac{x^3 - 2x^2 + 5}{3}</math> <span style="float: right;"><math>\left(y' = \frac{3x^2 - 4x}{3}\right)</math></span></p>   |
| <p>5. <math>y = -x</math> <span style="float: right;"><math>(y'=-1)</math></span></p>   | <p>24. <math>y = \sqrt{x}</math> <span style="float: right;"><math>\left(y' = \frac{1}{2\sqrt{x}}\right)</math></span></p>   |
| <p>6. <math>y = \frac{x}{2} - 5</math> <span style="float: right;"><math>(y'=1/2)</math></span></p>   | <p>25. <math>y = \sqrt{6x}</math> <span style="float: right;"><math>\left(y' = \frac{3}{\sqrt{6x}}\right)</math></span></p>  |
| <p>7. <math>y = x^4</math> <span style="float: right;"><math>(y'=4x^3)</math></span></p>  | <p>26. <math>y = \sqrt{x^2 + x + 1}</math> <span style="float: right;"><math>\left(y' = \frac{2x + 1}{2\sqrt{x^2 + x + 1}}\right)</math></span></p>  |
| <p>8. <math>y = 2x^5</math> <span style="float: right;"><math>(y'=10x^4)</math></span></p>  | <p>27. <math>y = \sqrt[3]{x}</math> <span style="float: right;"><math>\left(y' = \frac{1}{3\sqrt[3]{x^2}}\right)</math></span></p>   |
| <p>9. <math>y = \frac{x^3}{2}</math> <span style="float: right;"><math>\left(y' = \frac{3x^2}{2}\right)</math></span></p>   | <p>28. <math>y = \sqrt[3]{x^2}</math> <span style="float: right;"><math>\left(y' = \frac{2}{3\sqrt[3]{x}}\right)</math></span></p>   |
| <p>10. <math>y = x^3 + x^2 + x + 1</math> <span style="float: right;"><math>(y'=3x^2 + 2x + 1)</math></span></p>  | <p>29. <math>y = 2\sqrt[3]{x^4} - 3\sqrt{x+1}</math> <span style="float: right;"><math>\left(y' = \frac{8}{3}\sqrt[3]{x} - \frac{3}{2\sqrt{x+1}}\right)</math></span></p>                        |
| <p>11. <math>y = 2x^4 - 3x^2 + 5x - 8</math> <span style="float: right;"><math>(y'=8x^3 - 6x + 5)</math></span></p>   | <p>30. <math>y = (x^2 + 1)^2</math> <span style="float: right;"><math>(y'=4x^3 + 4x)</math></span></p>   |
| <p>12. <math>y = \frac{x^5}{5} - \frac{x^3}{3} + \frac{x^2}{4} - \frac{x}{7} + 5</math> <span style="float: right;"><math>\left(y' = x^4 - x^2 - \frac{x}{2} - \frac{1}{7}\right)</math></span></p> | <p>31. <math>y = (x^2 + 1)^{100}</math> <span style="float: right;"><math>(y'=200x(x^2 + 1)^{99})</math></span></p>  |
| <p>13. <math>y = -x^4 + \frac{1}{7}</math> <span style="float: right;"><math>(y'=-4x^3)</math></span></p>   | <p>32. <math>y = (2x^3 - 3x + 5)^3</math> <span style="float: right;"><math>(y'=3(2x^3 - 3x + 5)^2(6x^2 - 3))</math></span></p>  |
| <p>14. <math>y = \frac{1}{x}</math> <span style="float: right;"><math>\left(y' = -\frac{1}{x^2}\right)</math></span></p>  | <p>33. <math>y = 5(\sqrt{x} + 1)^2</math> <span style="float: right;"><math>\left(y' = \frac{5(\sqrt{x} + 1)}{\sqrt{x}}\right)</math></span></p>   |
| <p>15. <math>y = \frac{3}{x}</math> <span style="float: right;"><math>\left(y' = -\frac{3}{x^2}\right)</math></span></p>  | <p>34. <math>y = \left(x^2 + \frac{1}{x}\right)^5</math> <span style="float: right;"><math>\left(y' = 5\left(x^2 + \frac{1}{x}\right)^4 \left(2x - \frac{1}{x^2}\right)\right)</math></span></p> |
| <p>16. <math>y = \frac{1}{3x}</math> <span style="float: right;"><math>\left(y' = -\frac{1}{3x^2}\right)</math></span></p>  | <p>35. <math>y = (2x^2 - 3)(x^2 - 3x + 1)</math> <span style="float: right;"><math>(y'=8x^3 - 18x^2 - 2x + 9)</math></span></p>  |
| <p>17. <math>y = \frac{1}{x^2}</math> <span style="float: right;"><math>\left(y' = -\frac{2}{x^3}\right)</math></span></p>  |  |
| <p>18. <math>y = \frac{3}{x^3}</math> <span style="float: right;"><math>\left(y' = -\frac{9}{x^4}\right)</math></span></p>  |  |
| <p>19. <math>y = \frac{1}{2x^4}</math> <span style="float: right;"><math>\left(y' = -\frac{2}{x^5}\right)</math></span></p>   |  |

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| 36. $y = (x^2+x+1)(x^2-x+1)$       | $(y'=4x^3+2x)$   | 53. $y = \frac{1}{x\sqrt{x}}$      | $\left( y' = -\frac{3\sqrt{x}}{2x^3} \right)$                           |
| 37. $y = (x^2-3)(2x^2-5)^3$        |  | 54. $y = x^3 \sqrt{x}$             | $\left( y' = \frac{7\sqrt{x^5}}{2} \right)$                             |
| 38. $y = (x^2+1)(x-3)(x^2+x)$      | $(y'=5x^4-8x^3-6x^2-4x-3)$                                       | 55. $y = \frac{1}{(x^2+x+1)^2}$    | $\left( y' = -\frac{4x}{(x^2+x+1)^3} \right)$                           |
| 39. $y = x^2 \sqrt{x}$             | $\left( y' = \frac{5}{2} x \sqrt{x} \right)$                     | 56. $y = \frac{x}{x^2+1}$          | $\left( y' = -\frac{x^2+1}{(x^2+1)^2} \right)$                          |
| 40. $y = \sqrt[4]{x^3} (2x-3)$     | $\left( y' = \frac{14x-9}{4\sqrt[4]{x}} \right)$                 | 57. $y = \frac{x^2-1}{x^2+1}$      | $\left( y' = \frac{4x}{(x^2+1)^2} \right)$                              |
| 41. $y = \frac{2x-3}{2x+3}$        | $\left( y' = \frac{12}{(2x+3)^2} \right)$                        | 58. $y = \sqrt{\frac{x^2+1}{x+1}}$ | $\left( y' = \frac{(x^2+2x-1)\sqrt{x+1}}{2(x+1)^2\sqrt{x^2+1}} \right)$ |
| 42. $y = \frac{x^2-3}{2x+1}$       | $\left( y' = \frac{2x^2+2x+6}{(2x+1)^2} \right)$                 | 59. $y = \sqrt{\frac{x+1}{x-1}}$   | $\left( y' = -\frac{\sqrt{x-1}}{(x-1)^2\sqrt{x+1}} \right)$             |
| 43. $y = \frac{2x^2-1}{x^2+2}$     | $\left( y' = \frac{10x}{(x^2+2)^2} \right)$                      | 60. $y = \sqrt{x^5}$               | $\left( y' = \frac{5\sqrt{x^3}}{2} \right)$                             |
| 44. $y = \frac{3}{x^2-1}$          | $\left( y' = \frac{-6x}{(x^2-1)^2} \right)$                      | 61. $y = \frac{\sqrt{x+2}}{x^2}$   | $\left( y' = -\frac{3x+8}{2x^3\sqrt{x+2}} \right)$                      |
| 45. $y = \frac{x}{\sqrt{x}}$       | $\left( y' = \frac{1}{2\sqrt{x}} \right)$                        | 62. $y = \frac{2x+3}{x^2+4x-1}$    | $\left( y' = -\frac{2x^2+6x+14}{(x^2+4x-1)^2} \right)$                  |
| 46. $y = \sqrt{\frac{1}{x}+1}$     | $\left( y' = \frac{-1}{2x\sqrt{x^2+x}} \right)$                  | 63. $y = \frac{3x}{x^2-4}$         | $\left( y' = -\frac{3x^2+12}{(x^2-4)^2} \right)$                        |
| 47. $y = 3 \frac{x^2-4}{x^2+1}$    | $\left( y' = \frac{30x}{(x^2+1)^2} \right)$                      | 64. $y = \frac{x}{x-1}$            | $\left( y' = -\frac{1}{(x-1)^2} \right)$                                |
| 48. $y = \frac{(3x^2-1)^3}{x^2+1}$ | $\left( y' = \frac{108x^7+108x^5-108x^3+20x}{(x^2+1)^2} \right)$ | 65. $y = \sqrt{x^2-5}$             | $\left( y' = \frac{x}{\sqrt{x^2-5}} \right)$                            |
| 49. $y = \sqrt[4]{x^3}$            | $\left( y' = \frac{3}{4\sqrt[4]{x}} \right)$                     | 66. $y = x^6-10x^4+8x-3$           | $(y' = 6x^5-40x^3+8)$   |
| 50. $y = \frac{1}{\sqrt{x}}$       | $\left( y' = -\frac{\sqrt{x}}{2x^2} \right)$                     | 67. $y = \frac{x^3-x+1}{x-3}$      | $\left( y' = \frac{2x^3-9x^2+2}{(x-3)^2} \right)$                       |
| 51. $y = \frac{1}{\sqrt[3]{x}}$    | $\left( y' = \frac{-1}{3\sqrt[3]{x^4}} \right)$                  | 68. $y = \frac{x^2}{x^2-25}$       | $\left( y' = -\frac{50x}{(x^2-25)^2} \right)$                           |
| 52. $y = \frac{x}{\sqrt[3]{x}}$    | $\left( y' = \frac{-2}{3\sqrt[3]{x}} \right)$                    | 69. $y = 5x^4+x^3-x+6$             | $(y' = 20x^3+3x^2-1)$   |

70. $y = \sqrt[3]{2x^7}$	$\left( y' = \frac{7 \sqrt[3]{2x^7}}{3x} \right)$	76. $y = 4x + \sqrt[5]{x}$	$\left( y' = 4 + \frac{1}{5 \sqrt[5]{x^4}} \right)$
71. $y = \frac{5}{x} + \sqrt{x^3}$	$\left( y' = -\frac{5}{x^2} + \frac{3}{2} \sqrt{x} \right)$	77. $y = 5x + \frac{2}{x}$	$\left( y' = 5 - \frac{2}{x^2} \right)$
72. $y = \frac{x^2 + x - 2}{x + 1}$	$\left( y' = \frac{x^2 + 2x + 3}{(x + 1)^2} \right)$	78. $y = 5x^9 (3x + 2)^3$	$(y' = 45x^8 (3x + 2)^2 (4x + 2))$
73. $y = x^4 - 10x^2 + 8$	$(y' = 4x^3 - 20x)$	79. $y = \frac{x\sqrt{x}}{x + 2}$	$\left( y' = \frac{\sqrt{x}(x + 6)}{2(x + 2)^2} \right)$
74. $y = \sqrt[6]{x}$	$\left( y' = \frac{1}{6 \sqrt[6]{x^5}} \right)$	80. $y = \frac{2x}{5x + 8}$	$\left( y' = \frac{16}{(5x + 8)^2} \right)$
75. $y = \frac{5}{x^2} + \sqrt{x}$	$\left( y' = -\frac{10}{x^3} + \frac{1}{2\sqrt{x}} \right)$	81. $y = (x^3 + 8x)^{10}$	$(y' = 10 (x^3 + 8x)^9 (3x^2 + 8))$
		82. $y = \frac{3x - 1}{x^5 - 4x}$	$\left( y' = \frac{-12x^5 + 5x^4 - 4}{(x^5 - 4x)^2} \right)$

83. Deducir la fórmula de la derivada de  $y = \sqrt[n]{x}$  e  $y = \sqrt[n]{u}$

84. Deducir las derivadas de  $y = \frac{u}{v \cdot w}$  e  $y = \frac{u \cdot v}{w}$